

Exam. Code : 211004

Subject Code : 4297

M.Sc. (Mathematics) 4th Semester

STATISTICS—II

Paper—MATH-587

Time Allowed—Three Hours] [Maximum Marks—70

Note :— Attempt any **TWO** questions from each unit. All questions carry equal marks. A candidate may ask for statistical table.

UNIT—I

1. Define student-t statistic and describe its distribution. Find its variance and fourth order central moment μ_4 .
2. Describe the distribution of F-statistic and find its mean and variance.
3. Derive the sampling distribution of sample variance drawn from a normal distribution and find its mean.
4. A random sample of size n is drawn from an exponential population :

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta}, & x \geq 0, \theta > 0 \\ 0 & , \text{ otherwise} \end{cases}$$

Obtain the distribution of $X_{(r)}$. Also show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, $r < s$ are independently distributed.

UNIT—II

5. Explain what do you understand by sufficiency and completeness. Show that the family of Poisson distribution is complete.
6. Discuss the term 'unbiasedness' and consistency as applied to an estimator. Examine the unbiasedness and consistency of sample mean (\bar{x}) as an estimator of λ , the parameter of a Poisson distribution.
7. Explain the principle of maximum likelihood method of estimation. For a random sample of size n from $N(\mu, \sigma^2)$, find the maximum likelihood estimates for μ and σ^2 simultaneously.
8. What is meant by efficiency of an estimator ? Also show that the minimum variance unbiased estimator of an estimable parametric function $\psi(\theta)$ if it exists, is unique.

UNIT—III

9. Distinguish between simple and composite hypothesis. If x_1, x_2, \dots, x_n is a random sample from $N(\mu, \sigma^2)$, obtain a test of testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ (μ unspecified) by the likelihood ratio method.
10. Find the most powerful test based on a single observation on the random variable X , for testing $H_0 : X$ has the $N(0, 1)$ distribution, against $H_1 : X$ has the probability density function :

$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, -\infty < x < \infty.$$

11. Let X_1, X_2, \dots, X_n be a set of n Bernoulli random variables with parameter θ , $0 < \theta < 1$. Obtain UMPU test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.
12. Write an explanatory note on any **two** of the following :
- Null hypothesis and alternative hypothesis.
 - Critical region and two types of errors.
 - Uniformly most powerful critical region and uniformly most powerful unbiased region.

UNIT—IV

13. The number of automobile accidents per week in a certain city were as follows :

12, 8, 4, 9, 6, 15, 10, 14, 2, 20.

Are these figures in agreement with the belief that accident conditions remained uniform during the 10 week period ?

14. Ten cartons are taken at random from an automatic filling machine. The mean net weight of 10 cartons is 11.802 and standard deviation is 0.1502. Does the sample mean differ significantly from the intended weight of 1202 ? Use a test at level of significance 0.05.
15. Explain F-test for equality of population variances. Applying this test, show that the following samples come from the same normal population :

Sample	Size	Sample Mean	Sum of squares of deviations from mean
1	10	15	90
2	12	14	108

16. Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same against that they are not at 5% level.

UNIT—V

17. What do you mean by residuals and errors in the theory of linear models ? Establish a relationship between residual sum of squares and error sum of squares for the case of full rank general linear model.
18. Define general linear model. Clearly bring out the difference between fixed, random and mixed effect models.
19. What is meant by analysis of variance ? What are its underlying assumptions ? Give a complete analysis of one-way classified data for fixed effect model.
20. If the general-linear model of full rank $Y = X\beta + e$ is such that $E(e) = 0$ and $E(ee') = \sigma^2 I$. Then the best linear unbiased estimate of β is $S^{-1}X'Y$, where $S = X'X$. Explain it clearly.